Key

**Calculus BC: 4-1 Extreme Values of a Function**

1. Let $y = f(x) = x^3 - 12x - 5$

$f'(x) = 3x^2 - 12$

$3(x^2 - 4)$

$x = \pm 2$

Find the Absolute extrema for:

a. $x \in [0,5]$

Abs max $(5, 60)$

Abs min $(2, -21)$

C.I.T.

| $x$ | $f(x)$ |
|---|---|
| 0 | -5 |
| 2 | -21 |
| 5 | 60 |

b. $x \in [-5,5]$

Abs max $(5, 60)$

Abs min $(-5, -70)$

C.I.T.

| $x$ | $f(x)$ |
|---|---|
| -5 | -70 |
| -2 | 11 |
| 2 | -21 |
| 5 | 60 |

c. $x \in [-2,3]$

Abs max $(-2, 11)$

Abs min $(2, -21)$

C.I.T.

| $x$ | $f(x)$ |
|---|---|
| -2 | 11 |
| 2 | -21 |
| 3 | -14 |

**Conclusion:**

The absolute extrema for a function continuous on a **closed** interval will occur either at the endpoints or the critical points.

Critical Points: where $f'(x) = 0$ or is undefined

**Practice:**

1. For $g(x) = 2x^3 - 3x^2$ in $x \in [-1, 2]$

C I T

| $x$ | $g(x)$ |
|---|---|
| $-1$ | $-5$ |
| $0$ | $0$ |
| $1$ | $-1$ |
| $2$ | $4$ |

Abs min $(-1,-5)$

Abs Max $(2,4)$

local min $(1,-1)$

a. Find the absolute max and absolute min points.

$g'(x) = 6x^2 - 6x$

$= 6x(x-1)$

$x = 0, 1$

b. Find the inflection point of $g$.

$g''(x) = 12x - 6$

$x = \frac{1}{2}$

$\boxed{\left(\frac{1}{2}, -\frac{1}{2}\right)}$

* need sign change in $g''(x)$

2. Let $f(x) = x^3 - 6x^2 + 9x$ and $h(x) = 4$

a. Find the coordinates of the points common to $f$ and $h$.

$x^3 - 6x^2 + 9x - 4 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 9 & -4 \\ & & 1 & -5 & 4 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4, \quad x = 1$

$(x-1)^2(x-4)$

Points

$(1,4), (4,4)$

b. If the domain of $f$ is $x \in [0, 2]$, what is the range of $f$.

$f'(x) = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x-3)(x-1)$

C I T

| $x$ | $f(x)$ |
|---|---|
| 0 | 0 |
| 1 | 4 |
| 2 | 2 |

$\boxed{0 \le y \le 4}$

3. Determine $a$, $b$, c, and $d$ so that $f(x) = ax^3 + bx^2 + cx + d$ has a point of inflection at the origin and a relative max at the point $(2, 4)$.

$f(0) = 0 + 0 + 0 + d = 0$

③ $d = 0$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f'(2) = 0$

① $12a + 4b + c = 0$

$12a + c = 0$

$f''(0) = 0$

$0 + 2b = 0$

② $b = 0$

④ $f(2) = 8a + 2c = 4$ *

$12\left(-\frac{1}{4}\right) + c = 0$

$c = 3$

$12a + c = 0$

$8a + 2c = 4$

$24a + 2c = 0$

$8a + 2c = 4$

$16a = -4$

$a = -\frac{1}{4}$

$a = -\frac{1}{4}$

$b = 0$

$c = 3$

$d = 0$